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(Residential Autonomous College affiliated to University of Calcutta)

M.A./M.Sc. SECOND SEMESTER EXAMINATION, MAY 2018

FIRST YEAR [BATCH 2017-19]

APPLIED CHEMISTRY

Date : 21/05/2018

Time : 11 am – 1 pm

Paper : VII [ENGINEERING MATHEMATICS]

Full Marks : 50

[Use a separate Answer Book for each Group]

Group - A

(Answer any five questions)

[5×5]

1. a) The trace of a square matrix A, denoted by $\text{tr}A$, is the sum of the principle diagonal elements of A.
If A & B are square matrices of the same order, prove that—
i) $\text{tr}A + \text{tr}B = \text{tr}(A + B)$ ii) $\text{tr}(BA) = \text{tr}(AB)$ [2+1]
b) State Cayley-Hamilton Theorem. [2]
2. If A & B be two symmetric matrices of the same order then AB is symmetric iff $AB = BA$. [5]
3. If $A = \begin{pmatrix} 1 & 0 & 1 \\ 3 & 4 & 5 \\ 2 & 3 & 4 \end{pmatrix}$, Find A^{-1} . [5]
4. Let A is a 3×3 real matrix having the eigen value 2, 3, 1. $\begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}$, $\begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}$, $\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$ are the eigen vectors of A
corresponding to the eigen values 2, 3, 1 respectively. Find the matrix A. [5]
5. a) Show that the function e^z satisfy Cauchy-Riemann equations. [2·5]
b) Show that $\frac{d}{dz} e^z = e^z$. [2·5]
6. Find the harmonic conjugate of $u(x, y) = xy$ and also find the analytic function $f(z)$ using u. [5]
7. Find the poles of the function $\frac{1}{(z^2 + 1)^2}$ and also calculate the residue on those points. [2+3]

Group - B

(Answer any five questions)

[5×5]

8. a) Use the definition of limit to establish $\lim_{x \rightarrow 3} \frac{x^2 - 9}{x - 3} = 6$. [2]
b) Use the definition of continuity to show that $f(x) = \frac{x}{x-1}$ is continuous at $x = 0$. [3]
9. a) Verify Roll's theorem for $f(x) = |x|$ in $[-1, 1]$. [3]
b) Evaluate $\lim_{x \rightarrow \infty} x^{\frac{1}{x}}$. [2]

10. Show that $\frac{x}{1+x} < \log(1+x) < x$ for any $x > 0$. [5]

11. a) Evaluate : $\lim_{(x,y) \rightarrow (0,0)} \frac{x^2 - y^2}{x^2 + y^2}$. [2]

b) Verify Euler's theorem for $f(x, y) = \tan^{-1}\left(\frac{x}{y}\right)$. [3]

12. Find extreme values for $f(x) = x^5 - 5x^4 + 5x^3 - 1$. [5]

13. a) Evaluate : $\int_0^{\infty} e^{-x^2} dx$. [3]

b) Evaluate : $\int_0^1 \frac{dx}{\sqrt{1-x^2}}$. [2]

14. Show that $f(x, y) = 4x^2y - y^2 - 8x^4$ has maximum at (0,0). [5]

15. Show that $\iint_R e^{y/x} dx dy = \frac{e-1}{2}$ where R is the triangle bounded by $y = x$, $y = 0$ and $x = 1$. [5]

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